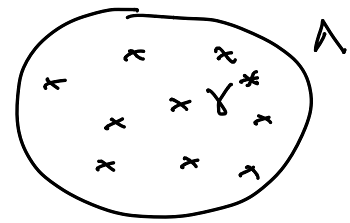
Lecture 3

Estimation of parameters

$$\mathbb{P}_\Lambda^{z,\beta} = \frac{1}{Z_\Lambda^{z,\beta}} e^{-\beta H} \pi_\Lambda^{z}$$



γ^* a realization of $P_\Lambda^{z^*,\beta^*}$ in Λ

z^*, β^* are unknown.

aim : $\hat{z}, \hat{\beta}$ estimators of z^*, β^*

extension : H^θ, $\theta \in \Theta$

- $H^\theta = \theta \cdot \mathbb{H} = \theta_1 H_1 + \dots + \theta_k H_k$ (linear case)
- non linear case

asymptotic : $\Lambda \to \mathbb{R}^d$

I) MLE

reformulation : $$P_\Lambda^{z,\beta} = \frac{1}{Z_\Lambda^{z,\beta}} e^{-\beta H} z^{N_\Lambda} \pi_\Lambda^{1}$$

Definition: MLE is definition

$$\hat{z}, \hat{\beta} = \underset{z,\beta}{\operatorname{argmax}} \frac{1}{Z_\Lambda^{z,\beta}} e^{-\beta H(\gamma^*)} z^{N_\Lambda(\gamma^*)}$$

Theorem with assumption assumptions. - Der - Lavancier

$$\hat{z}, \hat{\beta} \xrightarrow[\Lambda \to \mathbb{R}^d]{a.s} (z^*, \beta^*)$$

Remark: identifiability Ass $\iff$ Variational Principle.

$\ominus$. No result for normality

. untractable $Z_\Lambda^{z,\beta}$

$\oplus$. seems to the better

. efficient z^*, β^* are small (Mase 92)

II) Takacs-Fiksel Estimator (84)

GNZ equations.

$f : \mathbb{R}^d \times \Omega_f \longrightarrow \mathbb{R}^+$

$$C_\Lambda^{z,\beta}(f,\gamma) = \sum_{x \in \gamma_\Lambda} f(x, \gamma \setminus x) - z \int_\Lambda e^{-\beta h(x,\gamma)} f(x,\gamma)\, dx$$

$$\text{GNZ} \Longrightarrow \quad E_{P_\Lambda^{z^*,\beta^*}}\left(C_\Lambda^{z^*,\beta^*}(f, \Gamma) \right) = 0$$

TFE = mean square procedure

Definition $K \geq 2$, $(f_i)_{1 \leq i \leq K}$ K functions

$$(\hat{z}, \hat{\beta}) = \underset{(z,\beta)}{\text{argmin}} \sum_{k=1}^{K} \left(C_\Lambda^{z,\beta}(f_k, \gamma^*) \right)^2$$

Theorem. mild Ass

$(\hat{z}, \hat{\beta})$ is consistent and Ass Normal

(Coeurjolly, Der, Drouilhet, Lavancier

⊖ . identifiability Ass

. Variance is larger than MLE

. optimization problem

⊕ . Theoretical results

. flexible

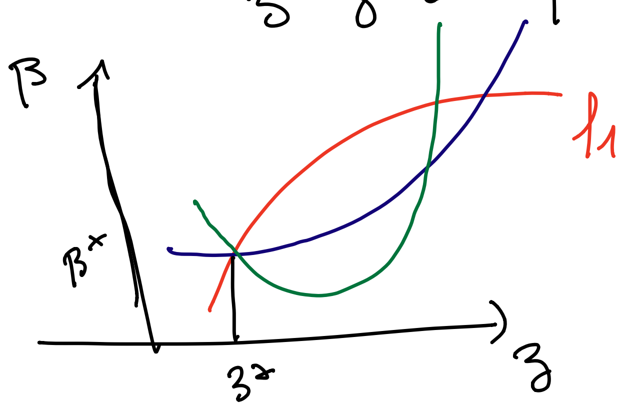
. no partition function .

<u>Identifiability Ass</u>:

$$\left(\mathbb{E}\left(f_k(0,\Gamma)\left[z e^{-\beta h(0,\Gamma)} - z^* e^{-\beta^* h(0,\Gamma)} \right] \right) = 0 \quad \forall k \right)$$

$$\Downarrow$$

$z = z^*$ and $\beta = \beta^*$



$\boxed{K = 3}$

<u>Sol</u>: $f_1(x,\gamma) = 1$, $f_2(x,\gamma) = h(x,\gamma)$ $\boxed{\text{Ok}}$

We recover the PMLE

Spatstat, R package Baddeley

IV) Variational Estimator (Baddeley Dec 2013)

<u>Variational GNZ equation</u>:

$$f : \Lambda \times \Omega_f \longrightarrow \mathbb{R}^+ \qquad (\text{i.e } f(x,\gamma))$$

we assume
- $\nabla_x f(x,\gamma)$ exists
- $f(x,\gamma) = 0$ for $x \in \partial\Lambda$
- $\nabla_x h(x,\gamma)$ exists

<u>Vectorial equation</u>:

$$\boxed{\mathbb{E}_{P_\Lambda^{z,\beta}}\left(\sum_{x\in\gamma_\Lambda} \nabla_x f(x,\gamma\backslash x)\right)}$$

$$(\mathrm{GNZ}) \quad = z\,\mathbb{E}_{P_\Lambda^{z,\beta}}\left(\int_\Lambda e^{-\beta h(x,\gamma)} \nabla_x f(x,\gamma)\,dx\right)$$

$$= z\beta\,\mathbb{E}_{P_\Lambda^{z,\beta}}\int_\Lambda \nabla h(x,\beta)\, e^{-\beta h(x,\gamma)} f(x,\gamma)\,dx$$

$$(\mathrm{GNZ}) \quad = \beta\,\mathbb{E}_{P_\Lambda^{z,\beta}}\left(\sum_{x\in\gamma_\Lambda} f(x,\gamma\backslash x)\nabla_x h(x,\gamma\backslash x)\,dx\right)$$

Definition The variational Estimator

$$\hat{\beta} = \frac{\sum_{x \in \gamma_\Lambda^*} \operatorname{div}_x f(x, \gamma^* \setminus x)}{\sum_{x \in \gamma_\Lambda^*} f(x, \gamma^* \setminus x) \operatorname{div}_x h(x, \gamma^* \setminus x)}$$

Theorem consistency and normality (Baddeley - Dereudre)

$$\boxed{f(x, \gamma) = \operatorname{div}_x f(x, \gamma)}$$

(−) . $\hat{z}$ is not estimated
. larger variance than the MLE and PMLE

(+) . Theoretical results
. flexible procedure
. simple to compute.